

# Schedule Compression by Fair Allocation Methods

by

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## 1. BACKGROUND

The IATA Worldwide Scheduling Guidelines describe a relatively orderly system in which airports move from one level to another as the number of scheduled operations approaches the airport capacity. Under the IATA system airports never reach a state of extreme congestion. Rather, as demand approaches capacity, slot controls are put in place in order to prevent such a situation from ever arising. Since the IATA system has not been used in the U.S., there are instances where airports have become highly congested to the extent that schedule compression is required to bring demand to a reasonable level. One can certainly argue that JFK airport is now in such a state. Before a solution is put in place, e.g. the use of the Worldwide Scheduling Guidelines, it is possible that other airports could reach such a state. If this happens then it would be necessary to reduce (perhaps significantly) the number of scheduled operations at one or more airports in order to effectively control congestion. Methods for doing this in an equitable manner are not specified as part of the IATA Guidelines. The purpose of this report is to outline an approach for doing this based on the application of certain principles from the body of knowledge on fair allocation methods (see Young [2]).

The specific situation we address would occur when the number of scheduled operations (arrivals and departures) at an airport exceeded the capacity of the airport. Here capacity would be specified by the number of arrivals and departures the runway system of the airport could sustain within a time window of specified length. Since flight operator schedules vary across the day, it would normally be the case that capacity might be exceeded only over limited time periods. For the methods described in this documents to be necessary it would have to be the case that capacity would be exceeded by a reasonably significant amount over several time windows. It is felt that the current status of JFK airport meets this criterion.

The output of the procedure would be a reduction in the number of scheduled operations so that demand was brought in line with capacity. Thus, for example suppose that the

number of scheduled operations between 8:00 and 9:00 AM was 91 but that the declared capacity was 80. Then the procedure would reduce the number of scheduled operations from 91 to 80. This reduction would be accomplished by specifying for each flight operator with scheduled operations in that time window, a reduced number of scheduled operations. Thus, for example, it could be the case that airline A had 10 scheduled operations and the procedure specified that these 10 must be reduced to 8. The procedure labels an operation as either an arrival or departure; the “other” origin or destination airport is irrelevant. Thus, after airline A’s operations were reduced from 10 to 8, airline A would be free to use those operations to access any other origins or destinations.

We now describe three key features of the procedure.

**Administrative procedure:** The procedure is based on the application of a fair resource allocation mechanism. It does not employ a market mechanism. At the same time, we should note that it determines an allocation when there is excess demand. It is not applicable in an obvious way in situations where new capacity becomes available. In such cases, we would strongly favor use of a market-based approach.

**Use of fair allocation principles:** The procedure draws on techniques from the body of knowledge on fair resource allocation. As such it requires a fair allocation standard, in order to determine what constitutes the most equitable allocation.

**Historical rights respected:** The allocation standard we recommend be used to guide the fair allocation process is historical airport usage. The use of historical rights makes this allocation process compatible with the IATA Guidelines. We should note that since the number of scheduled operations must be reduced, it is impossible to literally respect historical rights in the sense of insuring that each flight operator is able to maintain the exact number of operations that it has historically employed.

## 2. FAIR ALLOCATION PROCEDURE

The three major components of the procedure are i) a baseline schedule, ii) a revised level of operations and iii) a fair allocation procedure. The baseline schedule specifies a level of operations (arrivals and departures) for each carrier in each time window throughout the day (or week). The baseline schedule represents the standard to be used in determining what constitutes a fair allocation. That is, it is the historical schedule that represents the “grandfather rights” of the flight operators. There are several alternatives for choosing this to be discussed later. The revised level of operations is the reduced level of operations that the procedure will seek to achieve. Thus, for each time window it specifies a maximum level of operations, which will be lower than the current level in at least some time windows. The fair allocation procedure is the process that starts with the baseline schedule and removes operations from various flight operators so that the revised level of operations is achieved.

The principal goal of this report is to specify the fair allocation procedure. It can work with several alternative methods for specifying the baseline schedule and the revised level of operations. Before defining the fair allocation procedure, we first provide some thoughts on the baseline schedule and reduced level of operations.

## **2.1 Determining a Baseline Schedule**

The baseline schedule is an essential and very critical input to the process, as it represents the rights of each flight operator and directly determines the resources allocation that each flight operator receives. The most obvious approach would be to pick a “typical” week in the most recent heavily scheduled season and use the schedule for that week as the baseline. For example, in the case of the current activities related to JFK, a week from the summer of 2007 might be chosen. An alternative might be to choose a week from each of three or four recent scheduling seasons and to average the associated schedules in some way. For example, one week from each of January, July and October of 2007 might be chosen for this purpose. Finally, if it were desirable to give longer term history more weight, then a set of weeks from the past 5 years might be chosen and averaged in some way. This would give greater weight to carriers with schedule longevity. Each of these approaches has its own justification. Each constitutes an alternative policy option.

## **2.2 Determining a Revised Level of Operations**

The most typical approach to determining an airport capacity is to set a fixed hourly rate based on an assumption of close to ideal airport conditions. We would recommend a more judicious approach that takes into account the uncertainty associated with airport capacity (due to weather) and the uncertainty associated with flight operations, e.g. the fact that flights can be canceled and/or delayed for a variety of reasons not having to do with airport congestion. With this in mind, it can make sense to set the level of scheduled operations higher or lower than the declared capacity in certain time windows depending on the circumstances. Further, the best capacity profile might involve varying the number of scheduled operations over the course of the day. For example, when one takes into account that certain time windows have a higher value to flight operators (and passengers) than others, the most economic policy can be to schedule a slightly higher level of operations during the most desirable periods, while scheduling fewer flights during the less desirable periods. These periods of lower scheduled operations can serve as “cooling off” periods for high-delay days. The recent thesis of Churchill [1] presents a model for creating capacity profiles of the type just described.

## **2.3 The Fair Allocation Mechanism**

The procedure proceeds from one time window to another, iteratively adjusting the number of operations for each flight operator in the base schedule so that the capacity limit is respected and the revised level of operations is met. In this description we will use the generic term “operation.” As discussed later, this could refer to an arrival, a departure or either, depending on exactly how the process is set up. As a starting example, suppose that a time window currently had 17 scheduled operations but that the

revised level of operations indicated a capacity limit of 15. A fair allocation should proportionally reduce for each carrier,  $a$ , its current level of operations,  $O_a$ . Ideally carrier  $a$ 's revised level of operations should be  $15/17 O_a$ . Of course, in nearly all cases this approach would not work since  $15/17 O_a$  would not be an integer. Thus, some rounding procedure would need to be applied so that, typically, two carriers would have their level of operations reduced by 1, while all others receive no reduction. This process, of course, might be viewed as unfair, since two carriers would suffer a net loss, while all others would suffer no loss at all. The simple idea behind the fair allocation process is to keep a running tally of such net gains or losses and to take these into account when making decisions for future time windows. Thus, for example, if a flight operator received a net loss in one time window, then that flight operator should receive a net gain in another.

The following table illustrates three iterations of this process.

Airline		Base Ops		Share		Reduced Share		Allocated		Error
		<b>24</b>				<b>21</b>		21		
A	→	10	→	10	→	8.75	→	9	→	0.25
B	→	6	→	6	→	5.25	→	5	→	-0.25
C	→	2	→	2	→	1.75	→	2	→	0.25
D	→	6	→	6	→	5.25	→	5	→	-0.25
Airline		Base Ops		Share		Reduced Share		Allocated		Error
		<b>21</b>				<b>18</b>		18		
A	→	5	→	4.75	→	4.07	→	4	→	-0.07
B	→	4	→	4.25	→	3.64	→	4	→	0.36
C	→	3	→	2.75	→	2.36	→	2	→	-0.36
D	→	9	→	9.25	→	7.93	→	8	→	0.07
Airline		Base Ops		Share		Reduced Share		Allocated		Error
		<b>26</b>				<b>22</b>		22		
A	→	6	→	6.07	→	5.14	→	5	→	-0.14
B	→	3	→	2.64	→	2.24	→	2	→	-0.24
C	→	9	→	9.36	→	7.92	→	8	→	0.08
D	→	8	→	7.93	→	6.71	→	7	→	0.29

In this example, there are 4 airlines: A, B, C and D. The table illustrates the process of reducing operations in each of 3 time windows.

**Iteration (time window) #1:** In the first time window, the base level of operations is 24 and the reduced level is 21. Each of the 4 airlines starts respectively with 10, 6, 2 and 6 scheduled operations. The “ideal” reduced share for each airline is obtained by multiplying the reduction factor ( $21/24$ ) by the initial share. Each of

these reduced shares is rounded to the nearest integer to obtain the allocated number of operations, i.e. 9, 5, 2 and 5. These values indicate that in this time window airlines A, B and D must reduce their level of operations by 1 and airline C has no required reduction. The Error column gives the difference between the Reduced Share column and the Allocated column. If this value is positive, then the corresponding airline received more than it was owed and if it is negative, then the corresponding airline received less than it was owed. These error values will be carried over to the next iteration (time window) so that the process can correct any loss or gain by adjusting future allocations.

**Iteration (time window) #2:** In the second time window, the base level of operations is 21 and the reduced level is 18. Note the adjustment in each airline's share that is applied going from the Base Ops column to the Share column. This adjustment is calculated by subtracting the Error column in the previous iteration from the Base Ops column. Thus, airlines that received less than their ideal share in the previous iteration are appropriately adjusted upward. The reduction factor (18/21) is now applied to the Share column to obtain the Reduced Share column. The process then proceeds as earlier to obtain the Allocated column. Thus, in this time window airline A, C and D must reduce their operations by 1 and airline B has no required reduction. Again a new Error column is calculated.

**Iteration (time window) #3:** In this time window, the base level of operations is 26 and the reduced level is 22. Again the Base Ops column is adjusted based on the Error column from the prior iteration to obtain the Share column. The process proceeds as earlier, multiplying the Share column by the reduction factor (22/26) to obtain the Reduced Share and finally rounding to obtain the Allocated column. The general approach should now be clear.

We should comment on the specific rounding procedure since it might not always be the case that rounding the values in the Reduced Share column to the nearest integer will produce a set of carrier allocations whose sum is the target total allocation. There are actually multiple approaches that could be used, however, the overall procedure is not too sensitive to the specific rounding approach used since the error term is carried over from iteration to iteration. The following approach is probably the most natural.

*Define  $RS_a$  to be the entry in the Reduced Share column for airline  $a$  and  $TR$  to be the total reduced share. The procedure will calculate  $A_a$ , the entry in the Allocated column for airline  $a$ . Note that  $\sum_a A_a$  should equal  $TR$ . We use  $\lfloor A \rfloor$  to denote the smallest integer less than or equal to  $A$ , i.e. the value obtained by rounding down any real number  $A$ .*

**Rounding Algorithm:**

**Step 1:** For all  $a$ , set  $A_a = \lfloor RS_a \rfloor$ . (Note that  $\sum_a A_a \leq TR$ ).

If  $\sum_a A_a = TR$  then stop; otherwise go to Step 2.

**Step 2:** Order the airlines according to decreasing value of  $RS_a - A_a$ ; break ties arbitrarily.

For each airline  $a$ , in order, set  $A_a = A_a + 1$  until  $\sum_a A_a = TR$ .

To illustrate how this process would work, let us consider the first iteration of the example given earlier. The initial RS vector is (8.75, 5.25, 1.75, 5.25). This is rounded down in Step 1 to obtain: (8, 5, 1, 5); note that  $\sum_a A_a = 19 < 21 = TR$ . The vector of values of  $RS_a - A_a$  is (.75, .25, .75, .25). In Step 2, we order the airlines by A, C, B, D. (note that there were two ties that were broken arbitrarily). Thus, in Step 2, we successively set  $A_A = 8 + 1 = 9$  and  $A_C = 1 + 1 = 2$ . At this point,  $\sum_a A_a = 21 = TR$  so we stop.

## 2.4 Specific Parameters and Features.

**Time window width:** The entire process operates on a time-window basis and, as such, the time window width is a key parameter. The choices generally considered fall within the range of 15 minutes to 1 hour. Narrower windows limit airline scheduling flexibility, while wider windows run the risk of leading to unbalanced schedules and congestion. For example, with a 1-hour window, one might see the majority of flights in the 7 AM to 8 AM time window scheduled close to 8 AM. This in turn would lead to congestion at this time and significant delays, which could even spill over into later time windows. We will not make a specific recommendation here but only indicate that it is important to insure reasonable balance via narrower time windows to avoid congestion.

**Allocation based on arrivals, departures or generic operations:** Thus, far we have defined the procedure as allocating operations. In fact, there are two basic alternatives with respect to what rights are allocated. Specifically, are flight operators given the right to schedule a generic operation (arrival or departure) or are they given specific arrival operation and departure operation rights? In the latter case, within each time window, a specific maximum number of arrivals and a specific maximum number of departures would be set. Flight operators would be allocated a specific number of arrival “slots” and a specific number of departure “slots.” Certainly, the entire process would be easier to manage if one simply worked with generic operations rather than with arrivals and departures. However, airport runway configurations and procedures generally cannot treat arrivals and departures in a completely interchangeable fashion. Thus, if generic operations were allocated, then it might be necessary to set the maximum number of operations at an artificially low level in order to achieve congestion mitigation goals.

The procedure as stated could be directly applied in the case of allocating generic operations. If specific arrival and departure operations were allocated then some additional controls would have to be put in place. In particular, the simplest direct way of handling this case would be to apply the procedure to arrivals and departures in separate, independent steps. The problem with such an approach is that a carrier might not receive a compatible set of arrival and departure slots. It is even possible, although fairly unlikely, that a flight operator would receive a different number of arrival and departure slots. A more likely, undesirable outcome would be to receive arrival and departure slots that could not easily be arranged into a set of compatible flights (and aircraft schedules). One approach to avoiding such outcomes would be to initially pair compatible arrivals and departures and then to use the fair allocation procedure to eliminate arrival-departure pairs. Thus, for example, after an initial pairing was determined, the procedure could be

executed for arrival slots and then, when each arrival was eliminated, the corresponding departure would be eliminated as well. There are certainly many alternatives for coming up with an initial pairing. A very natural approach would be to pair arrivals and departures based on a historical fleet schedule. That is, each arrival would be paired with the departure that succeeded it in a historical aircraft schedule. If such an approach were used then there would not be as great a control over the balance of departures as over arrivals. However, the departures would generally be reasonably well balanced as they would tend to be spaced similar distances from their paired arrivals.

***Daily vs. weekly slots:*** The historical policy used in the U.S. at the High Density Rule (HDR) airports was to allocate slots on a weekday and weekend basis. That is, a single “slot” gave the holder the right to schedule an operation within the specific time window on every weekday. On the other hand, the IATA guidelines define slots on a day to day basis, i.e. there are Monday slots, Tuesday slots, etc. Thus, a flight operator could conceivably operate a flight three days a week and would need to obtain three slots to do so. Under the U.S. HDR system, there was an implicit assumption that any weekday flight would be scheduled on every day of the week and so slots were allocated on that basis. The procedures described in this report could readily be applied to either the daily version of the problem or the weekday version. Because of the presence of many international carriers at JFK, who, at times, operate irregular schedules to some of the more distant destinations, it would appear that the IATA approach, i.e. defining slots for each day, is most appropriate.

***Moving operations to less congested time windows:*** It is possible that there would be time windows at a congested airport where scheduled operations are below the level specified in the revised level of operations. It is also possible that certain carriers might want to move some of their operations into such periods rather than simply eliminating them when schedule reductions are specified by the procedures described earlier. This could be accomplished in an orderly fashion within our fair allocation procedure. We propose to implement this feature (if desired) by allowing flight operators to dynamically participate in the fair allocation process. Specifically, a pool of available slots would be maintained and, whenever a flight operator “lost” a scheduled operation, that flight operator would be given the opportunity to choose from among the slots remaining in the pool.

### **3. NEW CAPACITY AND MARKET MECHANISMS**

The process we have described does not provide a means for accommodating new entrants or flight operators wishing to expand. Nor does it provide a mechanism for dealing with new capacity. We feel that both of these issues should be addressed using market-based approaches. Specifically, it is important to define property rights for the slots that are allocated as part of the process we have described and to allow secondary market trading (buy-sell). Many argue that the secondary market in the U.S. associated with the HDR slots has not operated in a satisfactory fashion. Specifically, in recent years, nearly all transactions associated with the LGA HDR slots have involved situations of bankruptcy or other types of financial distress. Some possible distortions in the market

might include competitive advantages to holding slots rather than selling them, or unwillingness of purchasers to pay a fair price when political means to acquire slots for free are available. In any event, we recommend that the Federal Government continue to explore steps that would provide more liquidity in these markets.

It is tempting to try to extend the procedures we have described to allocate new airport capacity. In fact, a fundamental component of our approach is the presence of a baseline schedule on which to base the allocation. In the case of new capacity no such baseline exists. While incumbent flight operators can potentially justify grandfather rights to airport access based on investments made in gates and other resources, no such justification exists with respect to new capacity. Thus, we feel there are strong arguments in favor of the use of market-based approaches, e.g. slot auctions, for allocating new capacity.

## **References**

- [1] Churchill, A., 2007 *Determining the Number of Slots to Submit to a Market Mechanism at a Single Airport*, MS Thesis, Department of Civil and Environmental Engineering, University of Maryland.
- [2] Young, H.P. 1994. *Equity in Theory and Practice*, Princeton University Press, Princeton, NJ.