

# **Technical Report: Categorization and Allocation of Limited Slots**

## **Addendum to Supplemental Notice of Proposed Rule Making: Congestion Management Rule for LaGuardia Airport Docket No. FAA-2006-25709, RIN 2120-AI70**

by

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This technical report provides a detailed description of the procedure to be used for the assignment of Limited Slots as required in the Supplemental Notice of Proposed Rulemaking (SNPRM): Congestion Management Rule for LaGuardia Airport, Docket No. FAA-2006-25709, RIN 2120-AI70. Each operating authorization defined under the LaGuardia order would be converted to a corresponding slot. Each such slot has an associated ½ hour time window and an associated air carrier. Under the proposed rule each such slot will be designated as either a Common Slot or a Limited Slot.

Under the proposed rule, 20% (approximately) of the Limited Slots would revert to the FAA on the effective date of the proposed rule. These slots are called 0-year slots. 20% (approximately) of the Limited Slots would revert to the FAA in each of the subsequent four years. These are called 1-year, 2-year, 3-year and 4-year slots depending on the year in which they revert to the FAA. As described in the SNPRM body, when a slot reverts to the FAA, it will either be retired in order to reduce congestion or reallocated via auction.

Thus, to carry out the proposed rule a procedure is required to:

- determine the number of Limited Slots assigned to each carrier;
- determine the ½ hour time window for each such Limited Slot;
- assign a reversion year to each such Limited Slot, i.e. to designate each Limited Slot as either a 0-year, 1-year, 2-year, 3-year or 4-year slot.

In the remainder of this technical report, the procedures for carrying out each of these three functions are described. First, a discussion on slots and their attributes is provided.

### **0. Defining Slots**

A slot gives a carrier the right to schedule an operation in a ½-hour time window. Thus, each slot has two key attributes: a ½-hour time window and a designated carrier. The

procedure described here assigns additional attributes to slots: a designation of Common vs Limited and for each Limited Slot the designation of a reversion year (0, 1, 2, 3 or 4).

The inputs to the procedure will be a list of slots with the first two attributes and the outputs will be a designation of the additional attributes. To aid in record keeping a unique identifier will be assigned to each slot so that they can be clearly identified and their assigned attributes tracked. Table 1 illustrates possible input information for a set of slots.

**Table 1:** In this example, there are two air carriers XX and YY. XX has 3 slots from 700 to 729, 1 from 730 to 759, 2 from 800 to 829 etc. YY has 1 from 700 to 729, 1 from 730 to 759, 2 from 800 to 829, etc. Even though all slots in a given time window are “equivalent” each slot is given a unique identifier for later manipulation.

700-729	730-759	800-829	830-859	900-929	930-959	1000-1029	1030-1059
XX1	XX4	XX5	XX7	XX9	XX10	YY9	XX13
XX2	YY2	XX6	XX8		XX11		XX14
XX3		YY3	YY5		XX12		YY10
YY1		YY4	YY6		YY8		
			YY7				

It should be noted that the performance of an airport in general will depend on the mix of arrival and departure operations. With this in mind it may be necessary for the FAA operationally to work with carriers to balance arrivals and departures. However, for the purposes of the longer-term scheduling rights afforded by this proposed rule, a slot shall remain generic and provide the right to schedule either an arrival or departure.

**1. Determining the Number of Limited Slots Assigned to Each Carrier**

The SNPRM body gives a nearly complete description of the procedure for this task. Specifically,

*Each carrier at LaGuardia would be awarded up to 20 Common Slots as their base.*

*Air Canada would be awarded an additional 22 Common Slots*

*Under Option 1:*

*10% of each carrier’s remaining slots would be designated as Limited Slots*

*Under Option 2:*

*20% of each carrier’s remaining slots would be designated as Limited Slots*

*In the above calculations, any rounding required would be to the nearest integer with .5 rounded up.*

The following examples illustrate the procedure:

**Example:**

Carrier XX has a total of 24 slots:

20 designated as Common.

4 remain:

under option 1, 10% of 4 is .4, this is rounded down to 0 → no Limited Slots, Carrier XX receives 24 Common Slots

under option 2, 20% of 4 is .8; this is rounded up to 1 → 1 Limited Slot, Carrier XX receives 23 Common Slots and 1 Limited Slot.

Carrier YY has a total of 55 slots:

20 designated as Common.

35 remain:

Under option 1, 10% of 35 is 3.5; this is rounded to 4 → 4 Limited Slots, Carrier YY receives 51 Common Slots and 4 Limited Slots.

Under option 2, 20% of 35 is 7; this is rounded to 7 → 7 Limited Slots, Carrier YY receives 48 Common Slots and 7 Limited Slots.

**2. Determining the ½-hour time window for each Limited Slot.**

The ½-hour time window for each carrier's Limited Slots will be designated in three steps. Step 1 would insure that the number of operations in each 1-hour time window was no more than 75 by designating as Limited Slots certain slots from 1-hour time windows with more than 75 slots. Step 2 would allow each carrier to designate the time windows for ½ of their Limited Slots. In Step 3, the FAA would designate the time windows for the remaining Limited Slots.

If N is the number of limited slots to be assigned to a particular carrier, then the time windows for N/2 slots would be designated by that carrier in Step 2 and the time windows for N/2 would be designated by the FAA using Steps 1 and 3. If N is odd then rounding is performed so that the number of limited slots designated by the FAA is one greater than the number designated by the carrier. For example, if carrier XX was

assigned 25 limited slots then the time windows for 22 slots would be designated by carrier XX and the time windows for 23 designated by the FAA. For the purposes of explaining the process in the remainder of this document  $FL(C)$  will represent the number of carrier C's slots to be assigned by the FAA. During the course of the procedure this array will be decremented as slots are assigned by the FAA so, when  $FL(C)$  equals 0 for the given carrier C, then no additional slots need to be designated for C.

***Step 1: Reducing Hourly Operations to 75.***

This step would only be executed under Option 1. For each 1-hour time window whose slot total exceeds 75, the number of slots that needed to be withdrawn to bring the total to 75 would be calculated. Calling the time window H and the number to be withdrawn W, the following procedure would then be applied:

*Until W slots have been withdrawn:*

*Remove from consideration in time window H, all slots belonging to any carrier, C, with  $FL(C) \leq 1$  or any carrier that has already been assigned a limited slot.*

*Choose randomly with equal likelihood a remaining slot in time window H.*

*Designate this slot as a 0-hour Limited Slot. Decrement  $FL(C)$  for the carrier C that owns the slot.*

Note that this procedure is designed so that no one carrier will be assigned more than one Limited Slot by this step. Also carriers to be assigned only one Limited Slot are not eligible.

***Step 2: Carrier-Designated Limited Slots***

In this step, each carrier would specify certain of its slots to be Limited Slots. As explained in the SNPRM body each carrier would be given a certain amount of time to communicate to the FAA the identifiers for these slots. The number to be specified is as explained at the beginning of Section 2.

***Step 3: FAA-Designated Limited Slots***

The step specifies the time window for the remaining Limited Slots. This step is divided into two phases. In phase 1, a quota is determined for each 1-hour time window. These quotas represent the number of Limited Slots to be specified for that time window in this step. In phase 2, actual slots are chosen so that the number of each chosen in each 1-hour time window exactly equals the respective quota specified in phase 1.

***Phase 1:*** The purpose of this phase is to specify the values in a array: QUOTA(H); here H is an index designating a 1-hour time window. The objective pursued in carrying out this task is to insure that, to the extent possible, Limited Slots are evenly distributed

throughout the day. This objective will be constrained by the time windows chosen by the carriers in Step 2. In fact, it can be anticipated that the time windows chosen in Step 2 will tend to heavily represent “less desirable” time periods. The SNPRM body indicates that the FAA will not place limited slots into time windows where the carriers have collectively assigned two or more limited slots. The two slot threshold represents a guideline that one can expect the solution to satisfy but will not be used as a strict requirement of the procedure, i.e. it is possible that depending on how the carriers distribute slots in Step 2, that the FAA may add additional slots to time windows with two (or more) carrier designated limited slots.

The procedure for calculating QUOTA(H) is now described. Two input parameters are required:

*F\_TOT*: the total number of Limited Arrival Slots to be assigned in Step 3 -- this is simply the total number of Limited Slots minus the number assigned in Step 1; it can also be obtained by summing the FL(C) over all carriers, C.

*AVAIL\_S(H)*: the number of slots currently (at this stage of the allocation process) subject to reassignment in each 1-hour time window H -- Some 1-hour time windows will have been assigned limited slots by carriers Step 2. Certain, 1-hour time windows will also have some historically unused slots. The sum of the number of limited slots assigned in Step 2 and the historically unused slots equals the number of slots currently available for reassignment. The objective of this phase is to set QUOTA(H) so that the total available slots, QUOTA(H) + AVAIL\_S(H) is approximately equal, i.e. within 1 of each other. We note that for those time windows to which the carriers collectively have assigned large numbers of slots, this will not be possible (these time windows will not be assigned any additional limited slots). However, for most slots this objective will be achieved.

The procedure also uses a parameter, K, which indicates the current minimum number of available slots in any time window. The following procedure determines QUOTA(H):

Set  $QUOTA(H) = 0$  for all 1-hour time windows, H, at LGA.

Set  $K = 0$

LOOP:

for each 1-hour time window H, if  $AVAIL_S(H) + QUOTA(H) = K$ , then set  $QUOTA(H) = QUOTA(H) + 1$ .

Set  $K = K + 1$ .

If  $\sum_H QUOTA(H) \geq F\_TOT$ , then go to FINISH.

Otherwise, repeat LOOP.

FINISH:

Until  $\sum_H QUOTA(H) = F\_TOT$  repeat the following procedure:

Consider all 1-hour time windows  $H$  with  $AVAIL\_S(H) + QUOTA(H) = K$  and  $QUOTA(H) > 0$ ; Randomly choose one such time window,  $H'$ , with equal likelihood. Set  $QUOTA(H') = QUOTA^A(H') - 1$ .

At the end of this process the array QUOTA will have been set. QUOTA(H) will equal the number of limited slots from time window H to be assigned to carriers in Phase 2.

**Example:**

For this example, limit the number of hours to four -- from 700-1059

Suppose there are four limited arrival slots to be assigned by FAA in these hours.

That is  $F\_TOT = 4$ .

Assume that carriers have assigned 2 limited arrival slots to the 700-759 hour and two limited arrival slots to the 1000-1059 hour; also assume that there is one unused slot in the 900-959 hour.

**Table 2:**

Limited Slots Assigned by Carriers + Unused Slots				
Hour	700-759	800-859	900-959	1000-1059
	XX1		UU1	YY2
	XX2			ZZ1
	YY1			ZZ2
AVAIL(H)	3	0	1	3

There are three slots that will be reallocated in the 700-759 hour, three slots that will be reallocated in the 1000-1059 hour and one slot that will be reallocated in the 900-959 hour. At this point, in the program, there are zero slots to be reallocated in the 900-959 hour. These are the AVAIL values for these hours at this point in the process.

Now assume that the FAA has not assigned any limited slots to any hour; that is the QUOTA(H) for each of the four hours is zero. Adding QUOTA(H) to AVAIL(H) for each hour, the following is obtained.

**Table 3: Status at Beginning of Step 1**

Hour	700-759	800-859	900-959	1000-1059
AVAIL(H)	3	0	1	3
QUOTA(H)	0	0	0	0
SUM	3	0	1	3

In the first iteration or step in the procedure  $K=0$ ; the

procedure then looks at the SUM (of AVAIL and QUOTA) in each hour, and if  $SUM = K=0$ , it will reset the QUOTA in that hour to 1 (i.e. it will add a limited slots in that hour). In Table 3 since the SUM is zero for the 800-859 hour, it is assigned a limited slot by resetting its QUOTA to 1. At the end of the first step, the status is as shown in Table 4.

**Table 4:** Status at End of Step 1

Hour	700-759	800-859	900-959	1000-1059
AVAIL(H)	3	0	1	3
QUOTA(H)	0	<b>1</b>	0	0
SUM	2	1	1	2

Notice that only one slot has been allocated which is less than the total number to be allocated ( $F\_TOT =4$ )

Table 4 then becomes the Status at the Beginning of Step 2

Now K is incremented by 1 so in Step 2,  $K=1$ . In Table 4, the SUM (of AVAIL and QUOTA) for both 800-859 and 900-959 equal 1 ( $=K$ ), so the QUOTA for both hours will be incremented by 1, which produces Table 5.

**Table 5:** Status at End of Step 2

Hour	700-759	800-859	900-959	1000-1059
AVAIL(H)	3	0	1	3
QUOTA(H)	0	<b>2</b>	<b>1</b>	0
SUM	3	2	2	3

The total number of slots reallocated (the sum of QUOTA over all hours) is now 3, which is less than the number to be allocated ( $F\_TOT=4$ ), so the procedure continues and Table 5 becomes the beginning of Step 3.

Now K is incremented by 1,so in Step 3,  $K=2$ . In Table 5, the SUM (of AVAIL and QUOTA) for both 800-859 and 900-959 equal 2 ( $=K$ ), so the QUOTA for both hours will be incremented by 1, which produces Table 6.

**Table 6:** Status at End of Step 3

Hour	700-759	800-859	900-959	1000-1059
AVAIL(H)	3	0	1	3
QUOTA(H)	0	<b>3</b>	<b>2</b>	0
SUM	3	3	3	3

The total number of slots reallocated (the sum of QUOTA over all hours) is now 5, which is greater than the number to be allocated ( $F\_TOT=4$ ), so the procedure stops allocating slots, and now must reduce QUOTA in one of the two hours (800-859 or 900-959) by one.

To do this, the procedure draws a random integer between 1 and 2. If it picks 1, then the QUOTA in 800-859 will be decremented by 1; if it picks two, Then QUOTA in 900-959 would be reduced by 1. Suppose the random number is 1, then the QUOTA in 800-859 is decremented by 1 and the final result is shown in Table 7

**Table 7:** Status at End of Phase 1

Hour	700-759	800-859	900-959	1000-1059
AVAIL(H)	3	0	1	3
QUOTA(H)	0	2	2	0
SUM	3	2	3	3

The total number of limited arrival slots to be allocated ( $F\_TOT$ ) equals the total QUOTA assigned to all hours equals 4 so, Phase 1 is complete.

**Phase 2:** In this Phase Limited Slots are assigned to carriers so that the number of Limited Slots assigned in any given 1-hour time window  $H$  equals  $QUOTA(H)$ . The following procedure is used:

*Order all remaining carriers by increasing value of  $FL(C)$  (ties are broken by drawing random numbers).*

*Repeat until  $QUOTA(H)=0$  for all 1-hour time windows  $H$ :*

*Eliminate from consideration any carriers  $C$  with  $FL(C) = 0$ ;*

*for each remaining carrier  $C$  in order:*

*Consider all of carrier  $C$ 's slots contained in some time window  $H$  with  $QUOTA(H)>0$ ; choose one such slot randomly with equal likelihood; designate that slot as a limited slot; decrement  $QUOTA(H)$  by 1 for the associated time window  $t$  and decrement  $FL(C)$  by 1. (This step will be executed in a way that spreads out the limited arrival slots assigned to a given carrier as much as possible. Specifically, each time this procedure is executed, the maximum number of limited arrival slots assigned to carrier  $c$  in any 1-hour time window will be calculated. The step will be executed so as not to increase this maximum value by eliminating 1-hour*

*time windows at the maximum value from consideration, if this can be done).*

It is theoretically possible that this procedure can “get stuck”. Specifically, it could be that when the procedure considers all 1-hour time windows with  $QUOTA(H) > 0$  that carrier C has no remaining slots in those time windows. Given the specific structure of the data (two very large carriers with operations scattered throughout the day), such a scenario is highly unlikely. In the unlikely event that this occurs, then Phase 2 will be repeated, starting with a different random number seed. Because this is a random process the manner in which the procedure progresses will be different and it is likely that the procedure will not “get stuck” a second time. If it did then the Phase 2 would again be re-executed. This process would be iterated until the procedure completed successfully. It is extremely unlikely that more than one iteration will be required.

### **Example:**

To illustrate the execution of the main iteration in the above procedure consider the input data in Table 1 and suppose that YY is the carrier under consideration. Further suppose that the slots designated by carriers in Step 2 and any Limited Slots previously designated by the FAA have already been removed. Thus, all the slots listed in Table 1 are eligible. Now suppose that  $QUOTA(800-859) = 0$  but that the other 1-hour time windows have  $QUOTA(H) > 0$ . Then, the eligible slots for YY are: YY1, YY2, YY8, YY9, YY10. Since there are 5 slots, a random number between 1 and 5 would be drawn. Suppose the result of this draw is 4. Then YY9 would be designated as a Limited Slot.

To illustrate the approach for “spreading out” the allocation for a given carrier, suppose further that YY has been allocated one Limited Slot in time window 1000-1059 but has not been allocated Limited Slots in any other time windows. Then the maximum number of Limited Slots in a time window is 1 and one time window has this maximum level (1000-1059). Thus, 1000-1059 is also eliminated from consideration and only slots in time windows 700-759 and 900-959 are considered giving: YY1, YY2, Y8. A random number between 1 and 3 would be drawn. Suppose the result of this draw is 2. Then YY2 would be designated a Limited Slot.

### **3. Assigning a Reversion Year to Each Limited Slot**

The Limited Slots designated in Step 1 were all specified to be 0-year slots. The reversion year for all other Limited Slots will be assigned by the procedure described in this Section.

Except for the slots assigned in Step 1, each carrier’s Limited Slots will be ordered as follows:

FAA designated (Step 3) slots precede carrier designated (Step 2) slots.

Within the list of FAA designated slots and the list of carrier designated slots, slots will be ordered by increasing slot time.

Each carrier will be considered in succession with the next carrier at each step being chosen randomly with equal likelihood.

When a carrier is considered, if that carrier has been assigned a 0-year slot in Step 1, then the first slot on the list will be assigned a reversion year of 4. If the number of remaining slots on the list is odd then the next slot on the list will be assigned a reversion year of 2. The remaining slots on the list will be assigned reversion years in pairs cycling through (1,3), (0,4) and (2,2). Each succeeding carrier will start in the next position in the cycle after the last pair assigned to the preceding carrier. The pair (2,2) or (0,4) may be dropped out of the final few assignments in order to balance the number of slots in each reversion year as much as possible.

**Example:**

In assigning the reversion years to carrier Limited Slots, the idea is that on average, each carrier’s limited slots will have leases that average approximately two years  $(0+1+2+3+4)/(5 \text{ auctions})$  To illustrate this procedure consider Table 8, which represents a possible result of the previous steps.

**Table 8:** C → Common Slot; L1 → Limited Slot designated in Step 1; L2 → Limited Slot designated in Step 2, i.e. a carrier designated slot; L3 → Limited Slot designated in Step 3.

700-729	730-759	800-829	830-859	900-929	930-959	1000-1029	1030-1059
XX1 C	XX4 C	XX5 C	XX7 L1	XX9 C	XX10 C	YY9 L2	XX13 L2
XX2 C	YY2 L2	XX6 C	XX8 C		XX11 L3		XX14 C
XX3 L2		YY3 L3	YY5 C		XX12 L3		YY10 C
YY1 C		YY4 C	YY6 L3		YY8 C		
			YY7 C				

The process iterates through all carriers in random order. In Table 8 there are two carriers, so a random number between 1 and 2 would be drawn. Suppose the draw is 1 and carriers are considered alphabetically so that 1 corresponds to XX.

Processing for XX:

XX has one slot assigned in Step 1, XX7. This would be designated a 0-year slot. Beginning with slots designated in Step 3 and working back the Step 2, the order of the remaining Limited Slots is:

XXX11, XX12, XX3, XX13.

Since XX had a slot assigned in Step 1, the first slot on the list would be assigned a reversion year of 4. Since the number of remaining slots is odd, the next slot would be assigned a reversion year of 2. The process starts cycling through the pairs with (1,3) so XX3 becomes a 1-year slot and XX13 a 3-year slot.

Processing for YY:

Since YY is the final carrier no additional random draw is necessary to choose the next carrier.

YY has no slot assigned in Step 1; its slot order (beginning with Step 3) is:

YY3, YY6, YY2, YY9.

Since the number of slots is even, the process immediately starts by cycling through the pairs. It starts with the pair following the one used last in the prior iteration. The process starts with (0,4) and ends with (2,2) so that YY3 becomes a 0-year slot, YY6 a 4-year slot and YY2 and YY9 become 2-year slots.